NAVAL POSTGRADUATE SCHOOL
Monterey, California

THESIS

FINITE ELEMENT MODELING OF PARTIALLY DELAMINATED COMPOSITE BEAMS WITH CONTACT-IMPACT CONDITIONS

by

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June, 1994

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REPORT DOCUMENTATION PAGE	Form Approved OMB No. 0704-0188

1. AGENCY USE ONLY	2. REPORT DATE June 1994	3. REPORT TYPE AND DATES COVERED Master's Thesis

4. TITLE AND SUBTITLE FINITE ELEMENT MODELING OF PARTIALLY DELAMINATED COMPOSITE BEAMS WITH CONTACT-IMPACT CONDITIONS (UNCLASSIFIED)

5. FUNDING NUMBERS

6. AUTHOR(S) Haluk Aygunes

7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)
Naval Postgraduate School
Monterey CA 93943-5000

8. PERFORMING ORGANIZATION REPORT NUMBER

9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)

10. SPONSORING/MONITORING AGENCY REPORT NUMBER

11. SUPPLEMENTARY NOTES The views expressed in this thesis are those of the author and do not reflect the official policy or position of the Department of Defense or the U.S. Government.

12a. DISTRIBUTION/AVAILABILITY STATEMENT
Approved for public release; distribution is unlimited.

12b. DISTRIBUTION CODE
*A

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14. SUBJECT TERMS FINITE ELEMENT METHOD, DELAMINATED COMPOSITE BEAM, CONTACT-IMPACT CONDITION	15. NUMBER OF PAGES 50
	16. PRICE CODE

17. SECURITY CLASSIFICATION OF REPORT	18. SECURITY CLASSIFICATION OF THIS PAGE	19. SECURITY CLASSIFICATION OF ABSTRACT	20. LIMITATION OF ABSTRACT
Unclassified	Unclassified	Unclassified	UL

NSN 7540-01-280-5500

Standard Form 298 (Rev. 2-89)
Prescribed by ANSI Std. 239-18

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Finite Element Modeling of Partially Delaminated Composite Beams with Contact-Impact Conditions

by

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Submitted in partial fulfillment
of the requirements for the degree of

MASTER OF SCIENCE IN MECHANICAL ENGINEERING

from the

NAVAL POSTGRADUATE SCHOOL
June 1994

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ACKNOWLEDGEMENTS

I would like to express my thanks to Professor Young W.Kwon for his guidence and technical support throughout this study.

I also would like to thank to my wife, Serap, for her never ending support and encouragement.

I. INTRODUCTION

Laminated composite structures provide improved mechanical properties such as high strength, stiffness, toughness and high temperature performance [Ref. 1]. However, these structures have more complicated failure mechanisms and modes compared to those of conventional metallic structures. One of the common failure mode is inter-delamination.

The objectives of this study are to develop a finite element model for an analysis of laminated composite beam structures containing an inter-delamination crack and to find a methodology to detect an embedded interlaminar delamination. For delaminated beams, a short impulse load is applied to find out whether the dynamic response varies significantly compared to the dynamic response of structures without a crack.

In this study, a newly developed finite element model [Ref. 2] is used for the simulation of beams containing an inter-delamination crack. A rectangular element that has six degrees of freedom is used. The element allows axial displacements at its four corner points and lateral displacements at its two ends. Using this element, static and dynamic analyses of laminated composite beams containing a local delamination crack are conducted. First of all, static deflections as well as natural frequencies and mode shapes of beams without delamination are calculated to check the accuracy of the element. Laminated composite beams with a partial delamination are studied using the contact-impact conditions applied at the delaminated portion.

II. FINITE ELEMENT FORMULATION

A. FINITE ELEMENT MODELING

A newly developed rectangular finite element, Figure 1, is used to investigate the static and dynamic behavior of laminated composite beams. The element has six degrees of freedom and allows axial and lateral displacements but no rotation. There are axial displacements at the four corner points and lateral displacements at the two ends of the element.

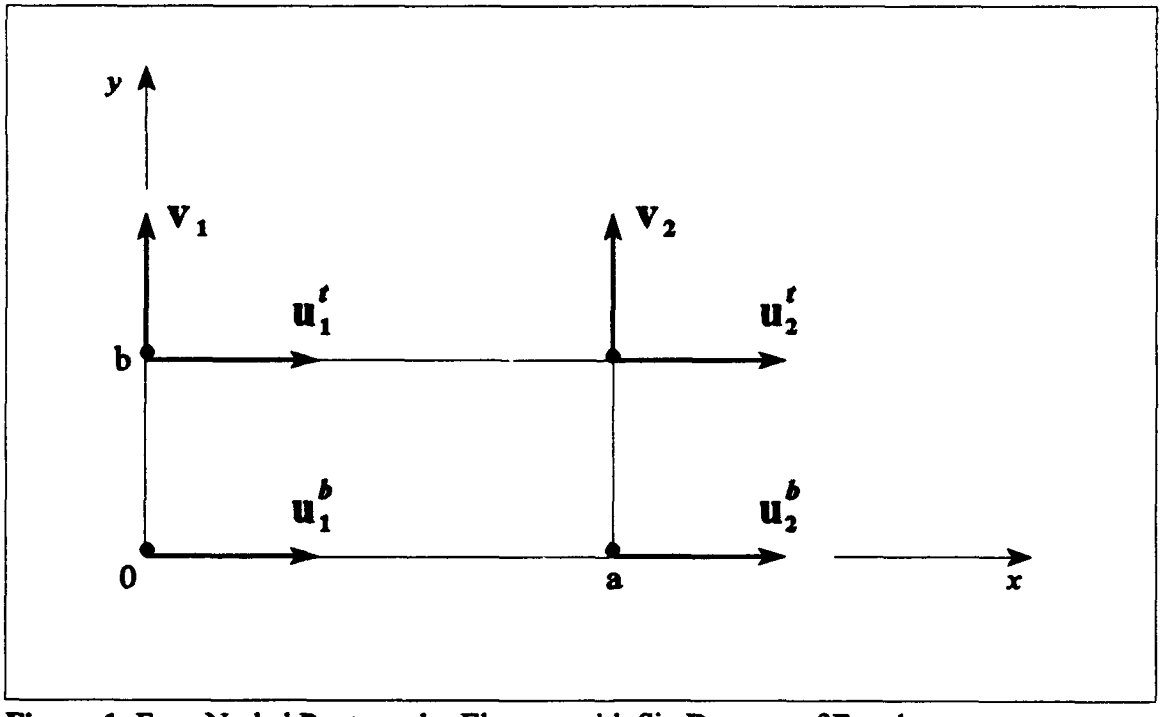

Figure 1 :Four Noded Rectangular Element with Six Degrees of Freedom.

In Figure 1 and the subsequent formulation, **u** represents the axial displacements at the corner points and **v** represents the lateral displacements at the ends. The subscripts "1" and "2" refer to the left and right ends while the superscripts "t" and "b" indicate the top and bottom sides of the element, respectively.

The displacement model of the element is

$$\bar{u}=\begin{Bmatrix} u(x,y) \\ v(x) \end{Bmatrix}=[N]\{d^{e}\} \tag{1}$$

where [N] is the matrix of shape functions and $\{d^e\}$ is a vector of nodal displacements. The axial displacement is assumed to vary linearly along both axial and lateral directions. It can be written as

$$\begin{aligned} u(x,y) &= \sum_{i=1}^{2} N_i(x)\,[H_1(y)\,u_i^b + H_2(y)\,u_i^t] \\ &= N_1(x)\,H_1(y)\,u_1^b + N_1(x)\,H_2(y)\,u_1^t + N_2(x)\,H_1(y)\,u_2^b + N_2(x)\,H_2(y)\,u_2^t \end{aligned} \tag{2}$$

The lateral displacement, which is assumed to be constant through the thickness of the element, varies linearly along the axial direction and can be written as

$$\begin{aligned} v(x) &= \sum_{i=1}^{2} N_i(x)\,v_i \\ &= N_1(x)\,v_1 + N_2(x)\,v_2 \end{aligned} \tag{3}$$

Here, N_i and H_i are the linear interpolation or shape functions in the axial and lateral directions. The beam element may use a higher order shape function for the axial direction, i.e. N_i, and the linear shape function for the lateral direction, i.e. H_i. However, in this study, the linear shape function is used for both N_i and Hi for simplicity. That is,

$$
\begin{aligned}
N_1(x) &= 1-\frac{x}{a} \\
N_2(x) &= \frac{x}{a} \\
H_1(y) &= 1-\frac{y}{b} \\
H_2(y) &= \frac{y}{b}
\end{aligned}
\tag{4}
$$

where a and b are the length and height of the beam element, respectively.

For simplicity, notations N_1, N_2, H_1 and H_2 will be used instead of $N_1(x)$, $N_2(x)$, $H_1(y)$ and $H_2(y)$ in the following derivation.

For two dimensional elasticity problems the strain vector can be expressed as [Ref. 3]

$$
\vec{\varepsilon} = \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} = \begin{bmatrix} \partial/\partial x & 0 \\ 0 & \partial/\partial y \\ \partial/\partial y & \partial/\partial x \end{bmatrix} \begin{Bmatrix} u \\ v \end{Bmatrix} = [B]\{d^e\}
\tag{5}
$$

where $\{d^e\}$ is the vector of nodal displacements and [B] is the matrix that relates strains to the nodal displacement vector. Normal strains can be written as

$$
\begin{aligned}
\varepsilon_x &= \frac{\partial u}{\partial x} = \frac{\partial N_1}{\partial x} H_1 u_1^b + \frac{\partial N_1}{\partial x} H_2 u_1^t + \frac{\partial N_2}{\partial x} H_1 u_2^b + \frac{\partial N_2}{\partial x} H_2 u_2^t \\
\varepsilon_y &= \frac{\partial v}{\partial y} = 0
\end{aligned}
\tag{6}
$$

and the shear strain is

$$\gamma_{xy}=\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}=\frac{\partial H_1}{\partial y}N_1 u_1^{\,b}+\frac{\partial H_2}{\partial y}N_1 u_1^{\,t}+\frac{\partial H_1}{\partial y}N_2 u_2^{\,b}+\frac{\partial H_2}{\partial x}N_2 u_2^{\,t}+\frac{\partial N_1}{\partial x}v_1+\frac{\partial N_2}{\partial x}v_2 \tag{7}$$

The element stiffness matrix can be obtained by minimizing the total strain energy which contains both bending and the transverse shear energy. This minimization yields the following element stiffness matrix

$$[k^e]=[k_B^e]+[k_S^e] \tag{8}$$

where the subscripts "B" and "S" indicates bending and the transverse shear respectively.

The bending and transverse shear stiffness matrices are given as

$$[k_B^e]=\int_0^a\int_0^b\{B_B\}^T E\{B_B\}\,dy\,dx \tag{9}$$

$$[k_S^e]=\int_0^a\int_0^b\{B_S\}^T G\{B_S\}\,dy\,dx \tag{10}$$

where E and G are the elastic and shear modulii of the beam and the vectors $\{B_B\}$ and $\{B_S\}$ are derived below.

The strain-displacement relationship for the normal strain in the axial direction is

$$\varepsilon_x = \{B_B\} \begin{Bmatrix} u_1^b \\ u_1^t \\ v_1 \\ u_2^b \\ u_2^t \\ v_2 \end{Bmatrix} \tag{11}$$

and the relationship for the shear strain is

$$\gamma_{xy} = \{B_S\} \begin{Bmatrix} u_1^b \\ u_1^t \\ v_1 \\ u_2^b \\ u_2^t \\ v_2 \end{Bmatrix} \tag{12}$$

Equations (11) and (12) yield

$$\begin{gathered} \{B_B\} = \left\{ \frac{\partial N_1}{\partial x} H_1 \quad \frac{\partial N_1}{\partial x} H_2 \quad 0 \quad \frac{\partial N_2}{\partial x} H_1 \quad \frac{\partial N_2}{\partial x} H_2 \quad 0 \right\} \\ \{B_S\} = \left\{ N_1 \frac{\partial H_1}{\partial y} \quad N_1 \frac{\partial H_2}{\partial y} \quad \frac{\partial N_1}{\partial x} \quad N_2 \frac{\partial H_1}{\partial y} \quad N_2 \frac{\partial H_2}{\partial y} \quad \frac{\partial N_2}{\partial x} \right\} \end{gathered} \tag{13}$$

The bending stiffness matrix can be obtained by carrying out the integration in Eq. (9) which will result in

$$[k_B^e] = \begin{bmatrix} \frac{Eb}{3a} & \frac{Eb}{6a} & 0 & -\frac{Eb}{3a} & -\frac{Eb}{6a} & 0 \\ \frac{Eb}{6a} & \frac{Eb}{3a} & 0 & -\frac{Eb}{6a} & -\frac{Eb}{3a} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ -\frac{Eb}{3a} & -\frac{Eb}{6a} & 0 & \frac{Eb}{3a} & \frac{Eb}{6a} & 0 \\ -\frac{Eb}{6a} & -\frac{Eb}{3a} & 0 & \frac{Eb}{6a} & \frac{Eb}{3a} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{14}$$

For Eq. (10) the reduced integration technique is used along the x-axis to prevent shear locking which occurs when the ratio of beam length to beam thickness is large. This integration yields the transverse shear stiffness matrix of the form

$$[k_s^e]=\begin{bmatrix} \frac{Ga}{4b} & -\frac{Ga}{4b} & \frac{G}{2} & \frac{Ga}{4b} & -\frac{Ga}{4b} & -\frac{G}{2} \\ -\frac{Ga}{4b} & \frac{Ga}{4b} & -\frac{G}{2} & -\frac{Ga}{4b} & \frac{Ga}{4b} & \frac{G}{2} \\ \frac{G}{2} & -\frac{G}{2} & \frac{Gb}{a} & \frac{G}{2} & -\frac{G}{2} & -\frac{Gb}{a} \\ \frac{Ga}{4b} & -\frac{Ga}{4b} & \frac{G}{2} & \frac{Ga}{4b} & -\frac{Ga}{4b} & -\frac{G}{2} \\ -\frac{Ga}{4b} & \frac{Ga}{4b} & -\frac{G}{2} & -\frac{Ga}{4b} & \frac{Ga}{4b} & \frac{G}{2} \\ -\frac{G}{2} & \frac{G}{2} & -\frac{Gb}{a} & -\frac{G}{2} & \frac{G}{2} & \frac{Gb}{a} \end{bmatrix} \qquad (15)$$

The element stiffness matrix, which is obtained by adding the bending and transverse stiffness matrices, can be expressed in the following form

$$[k^e]=\begin{bmatrix} a_1+2a_3 & -a_1+a_3 & a_4 & a_1-2a_3 & -a_1-a_3 & -a_4 \\ -a_1+a_3 & a_1+2a_3 & -a_4 & -a_1-a_3 & a_1-a_3 & a_4 \\ a_4 & -a_4 & a_2 & a_4 & -a_4 & -a_2 \\ a_1-2a_3 & -a_1-a_3 & a_4 & a_1+2a_3 & -a_1+a_3 & -a_4 \\ -a_1-a_3 & a_1-2a_3 & -a_4 & -a_1+a_3 & a_1+a_3 & a_4 \\ -a_4 & a_4 & -a_2 & -a_4 & a_4 & a_2 \end{bmatrix} \qquad (16)$$

where

$$a_1=\frac{Ga}{4b} \quad a_2=\frac{Gb}{a} \quad a_3=\frac{Eb}{6a} \quad a_4=\frac{G}{2} \qquad (17)$$

The mass matrix can be derived consistently in a similar way. In this study a diagonal (lumped) mass matrix was used since it takes less computing time. The mass matrix is

$$[m^e]=\rho ab\begin{bmatrix} \frac{1}{4} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{4} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{4} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{4} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{2} \end{bmatrix} \tag{18}$$

where ρ is the mass density, a and b are the length and height of the element. The element is assumed to have a unit depth.

B. LAMINATED COMPOSITE BEAMS

In the most generalized form Hooke's law can be written as

$$\sigma_{ij}=C_{ijkl}\varepsilon_{kl} \qquad i,j,k,l=1,2,3 \tag{19}$$

using tensor notations [Ref. 4]. Here C_{ijkl} represents the elastic stiffness. Using the symmetry of stress and strain tensors, i.e. $\sigma_{ij}=\sigma_{ji}$ and $\varepsilon_{ij}=\varepsilon_{ji}$, and using a shorthand notation we can write the normal stresses and strains as

$$\sigma_1=\sigma_{11} \qquad\qquad \varepsilon_1=\varepsilon_{11}$$

$\sigma_2=\sigma_{22}$ $\varepsilon_2=\varepsilon_{22}$

$\sigma_3=\sigma_{33}$ $\varepsilon_3=\varepsilon_{33}$

while the shear stresses and strains are written as

$\sigma_4=\sigma_{23}=\sigma_{32}$ $\varepsilon_4=\varepsilon_{23}=\varepsilon_{32}$

$\sigma_5=\sigma_{31}=\sigma_{13}$ $\varepsilon_5=\varepsilon_{31}=\varepsilon_{13}$

$\sigma_6=\sigma_{12}=\sigma_{21}$ $\varepsilon_6=\varepsilon_{12}=\varepsilon_{21}$

For general orthotropic materials Hooke's law can be written, in the inverted form, as

$$\begin{Bmatrix}\varepsilon_1\\\varepsilon_2\\\varepsilon_3\\\varepsilon_4\\\varepsilon_5\\\varepsilon_6\end{Bmatrix}=\begin{bmatrix}S_{11} & S_{12} & S_{13} & 0 & 0 & 0\\ & S_{22} & S_{23} & 0 & 0 & 0\\ & & S_{23} & 0 & 0 & 0\\ & & & S_{44} & 0 & 0\\ & & & & S_{55} & 0\\ & & & & & S_{66}\end{bmatrix}\begin{Bmatrix}\sigma_1\\\sigma_2\\\sigma_3\\\sigma_4\\\sigma_5\\\sigma_6\end{Bmatrix} \tag{20}$$

where $[S_{mn}]$ is the compliance matrix, which is the inverse of the matrix $[C_{mn}]$. For the state of plain stress, Eq. (20) is reduced to

$$\begin{Bmatrix}\varepsilon_1\\\varepsilon_2\\\varepsilon_6\end{Bmatrix}=\begin{bmatrix}S_{11} & S_{12} & 0\\ S_{12} & S_{22} & 0\\ 0 & 0 & S_{66}\end{bmatrix}\begin{Bmatrix}\sigma_1\\\sigma_2\\\sigma_6\end{Bmatrix} \tag{21}$$

Stress-strain relation can be obtained by inverting the relation given in Eq. (21)

$$\begin{Bmatrix}\sigma_1\\\sigma_2\\\sigma_6\end{Bmatrix}=\begin{bmatrix}Q_{11} & Q_{12} & 0\\ Q_{12} & Q_{22} & 0\\ 0 & 0 & Q_{66}\end{bmatrix}\begin{Bmatrix}\varepsilon_1\\\varepsilon_2\\\varepsilon_6\end{Bmatrix} \tag{22}$$

where the reduced stiffness terms are

$$
\begin{aligned}
Q_{11} &= \frac{E_1}{1-\nu_1\nu_2} \\
Q_{22} &= \frac{E_2}{1-\nu_1\nu_2} \\
Q_{12} &= \frac{\nu_1 E_1}{1-\nu_1\nu_2} = \frac{\nu_2 E_2}{1-\nu_1\nu_2} \\
Q_{66} &= G_6
\end{aligned}
\tag{23}
$$

In this study, the width of the beam is neglected, i.e. it is assumed to have a unit width. Therefore, only the elastic modulus along the beam axis is considered. Two different laminae with four and eight layers are used for calculations, i.e. $[0/90]_s$ and $[0/90/0/90]_s$ where the subscript "s" represents symmetry with respect to the middle axis of the beam.

There are several methods used in modeling laminated composite beams. In this study, two different techniques are used. The first technique discretizes respective layers in the finite element analysis. As a result, the total number of elements is proportional to the number of layers in a laminated beam. Of course, static condensation can be performed to reduce the number of total degrees of freedom by eliminating internal layers degrees of freedom. This modeling technique is computationally expensive but it can describe a general shape of deformation through the beam thickness. The second technique uses one beam element through the beam thickness regardless of the number of layers. This technique is computationally efficient but it assumes a linear deformation through the beam thickness. The development of this technique is described in the following paragraphs.

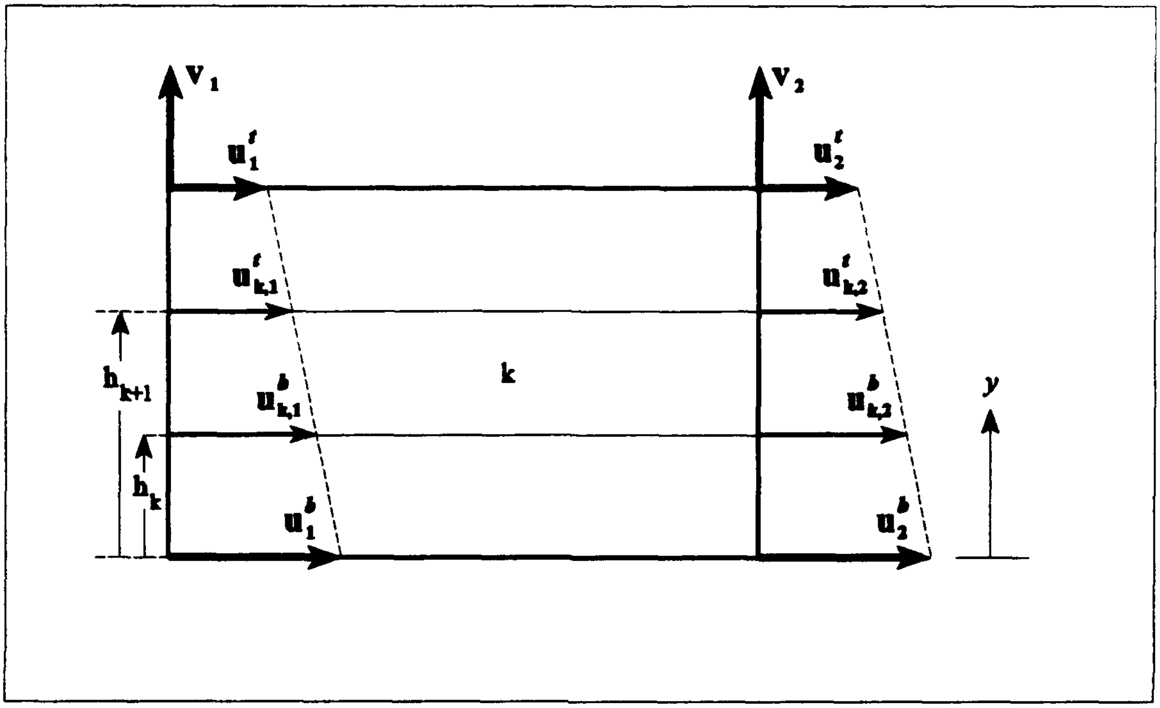

Figure 2 : Laminated Beam Element

Let $\mathbf{u}_k$ and $\mathbf{v}_k$ represent the axial and lateral displacements of the kth layer and let h be the beam thickness while h_k and h_{k+1} represent heights of the top and bottom sides of the layer measured from the bottom of the beam (see Figure 2). The relationship between the layer displacements and the global beam displacements can be written as

$$
\begin{aligned}
u_{k,1}^{b} &= c_1 u_1^{b} + c_2 u_1^{t} \\
u_{k,1}^{t} &= c_3 u_1^{b} + c_4 u_1^{t} \\
v_{k,1} &= v_1 \\
u_{k,2}^{b} &= c_1 u_1^{b} + c_2 u_1^{t} \\
u_{k,2}^{t} &= c_3 u_1^{b} + c_4 u_1^{t} \\
v_{k,2} &= v_2
\end{aligned}
\tag{24}
$$

where the constants, c's, are

$$
\begin{aligned}
c_1 &= \frac{h-h_k}{h} \\
c_2 &= \frac{h_k}{h} \\
c_3 &= \frac{h-h_{k+1}}{h} \\
c_4 &= \frac{h_{k+1}}{h}
\end{aligned}
\tag{25}
$$

This relationship can be expressed in the matrix form as

$$
\begin{Bmatrix} u_{k,1}^{b} \\ u_{k,1}^{t} \\ v_{k,1} \\ u_{k,2}^{b} \\ u_{k,2}^{t} \\ v_{k,2} \end{Bmatrix}
=
\begin{bmatrix}
c_1 & c_2 & 0 & 0 & 0 & 0 \\
c_3 & c_4 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & c_1 & c_2 & 0 \\
0 & 0 & 0 & c_3 & c_4 & 0 \\
0 & 0 & 0 & 0 & 0 & 1
\end{bmatrix}
\begin{Bmatrix} u_1^{b} \\ u_1^{t} \\ v_1 \\ u_2^{b} \\ u_2^{t} \\ v_2 \end{Bmatrix}
\tag{26}
$$

or

$$
\{d^k\} = [T]\{d\} \tag{27}
$$

where $\{d^k\}$ and $\{d\}$ are displacement vectors of *the* kth layer and the beam, and [T] is the transformation matrix. Now, stiffness and mass matrices of a laminated beam element can be expressed as

$$[k^*] = \sum_{k=1}^{N} [T]^T [k^k] [T]$$
$$[m^*] = \sum_{k=1}^{N} [T]^T [m^k] [T] \quad (28)$$

Here, $[k^k]$ and $[m^k]$ are the stiffness and mass matrices of the kth layer, respectively. Using this technique, a single beam element can include all the layers of a laminated beam.

C. DYNAMIC ANALYSIS

In dynamic analyses, displacements, velocities, accelerations, stresses and strains need to be calculated at each time interval since they all vary with time. This requires a time integration scheme. In this study, a form of central difference method is used in the dynamic analysis of undamped systems.

The general equation of motion at time t can be written as

$$[M]\{\ddot{u}\}_t + [K]\{u\}_t = \{F\}_t \quad (29)$$

The central difference method can be applied in three steps.

A) Compute [M] and [K]

B) Initial calculations

1. $\{\ddot{u}\}_0 = [M]^{-1}(\{F\}_0 - [K]\{u\}_0)$
2. $\{\dot{u}\}_{\frac{\Delta t}{2}} = \{\dot{u}\}_{-\frac{\Delta t}{2}} + \{\ddot{u}\}_0 (\Delta t)$ (30)
3. $\{u\}_{\Delta t} = \{u\}_0 + \{\dot{u}\}_{\frac{\Delta t}{2}} (\Delta t)$

C) For each time step

$$
\begin{aligned}
&1.\quad \{\ddot{u}\}_t = [M]^{-1}(\{F\}_t - [K]\{u\}_t) \\
&2.\quad \{\dot{u}\}_{t+\frac{\Delta t}{2}} = \{\dot{u}\}_{t-\frac{\Delta t}{2}} + \{\ddot{u}\}_t(\Delta t) \\
&3.\quad \{u\}_{t+\Delta t} = \{u\}_t + \{\dot{u}\}_{t+\frac{\Delta t}{2}}(\Delta t)
\end{aligned}
\tag{31}
$$

This method is conditionally stable when the time step size Δt is less than or equal to the critical step size [Ref. 5,6]

$$\Delta t \leq \Delta t_{cr} = \frac{2}{\omega_{max}} \tag{32}$$

where $(\omega_{max})^2$ is the largest eigenvalue of the matrix equation (19) with no forcing function. When $\Delta t > \Delta t_{cr}$ it results in an unstable numerical solution.

D. CONTACT-IMPACT PROBLEMS

In the case of partially delaminated beams, as shown in Figure 3, the upper and lower portions of the delaminated section may overlap each other if there is no constraint applied on the portion during the finite element calculations. This fact applies to both static and dynamic problems. To avoid this overlap the contact-impact conditions are applied.

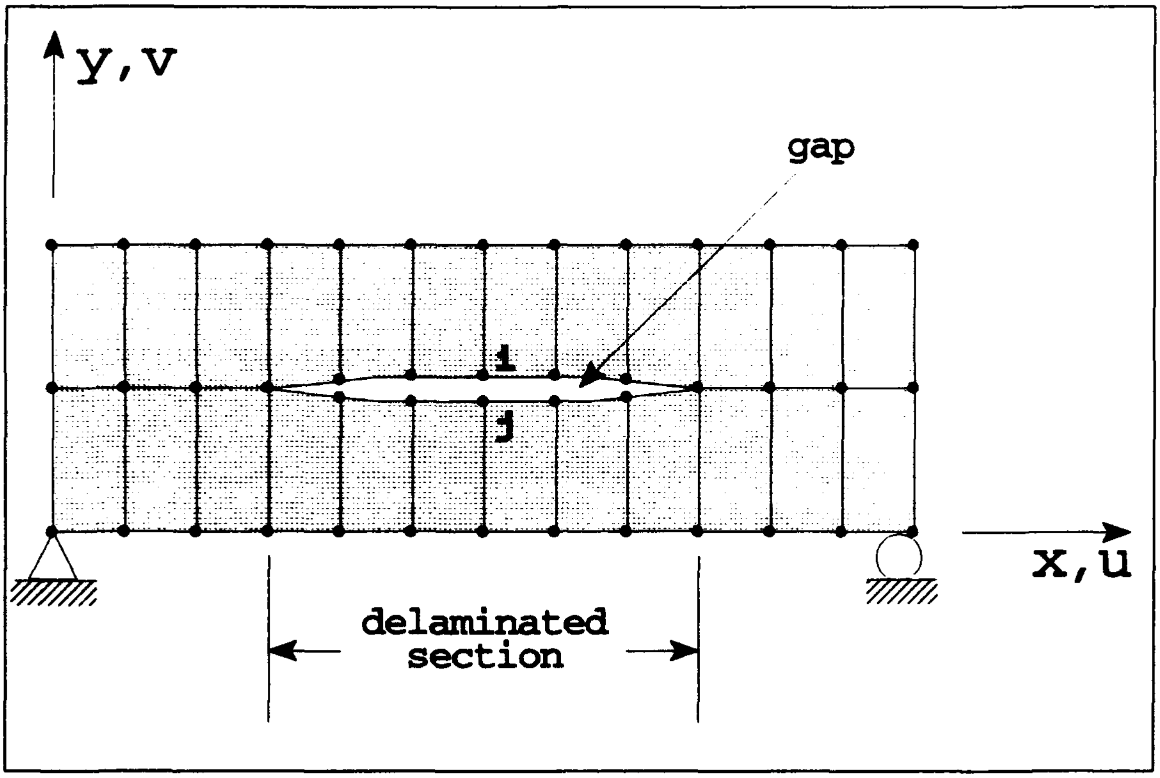

Figure 3 : Simply Supported Beam with a Delaminated Section.

The contact-impact algorithm developed in [Ref. 7] is applied to the present beam element as explained in the following paragraphs.

Let the letters i and j represent the facing nodal points in the upper and lower portions of a delaminated section, and let v_i and v_j represent the vertical displacements of these points. The procedure to apply the contact-impact condition is explained below.

A) Initially, the central difference method is used to calculate displacements, velocities and accelerations until any two points contact each other ($v_i-v_j>0$).

B) When the two nodes penetrate each other, i.e. $v_i-v_j \leq 0$, the release-to-contact condition is applied. And the corrected values of the velocity, acceleration and elastic force are calculated using the following formula;

$$\begin{aligned} \dot{u}_{+} &= \frac{M^1\dot{u}^1_{-1}+M^2\dot{u}^2_{-1}}{M^1+M^2} \\ \ddot{u}_{+} &= \frac{M^1\ddot{u}^1_{-}+M^2\ddot{u}^2_{-}}{M^1+M^2} \\ \tau_{+} &= \tau_{-}-\frac{M^1M^2}{M^1+M^2}(\ddot{u}^2_{-}-\ddot{u}^1_{-}) \end{aligned} \tag{33}$$

C) When they are released from each other, i.e. $v_i - v_j > 0$, the contact-to-release condition is applied and the corrected values of velocity and acceleration are computed using the following formula;

$$\begin{aligned} \dot{u}^{\alpha}_{+} &= \dot{u}_{-1}+\frac{(-1)^{\alpha}\Delta t\tau_{-1}}{2M^{\alpha}} \\ \ddot{u}^{\alpha}_{+} &= \ddot{u}^{\alpha}_{-}+\frac{(-1)^{\alpha}\tau_{-1}}{M^{\alpha}} \end{aligned} \tag{34}$$

where the value of α is 1 for the upper portion and 2 for the lower portion of the delaminated section. Here the subscript (+) is used for the updated values while the subscript (-1) indicates the values taken at the end of the previous time step and (-) indicates the values taken at the end of the last iteration of the present time step.

For static problems, the contact condition is applied simply by setting the vertical displacement of the upper portion equal to the vertical displacement of the lower portion where they overlap each other.

III. RESULTS AND DISCUSSION

A. VERIFICATION EXAMPLES

The present finite element model and the computer code are verified by analyzing three problems and comparing the results with exact solutions. The verification examples are static deflections of both isotropic and laminated composite beams and the free vibration of beams. Two different boundary conditions are applied. One of them is a simply supported beam and the other is a cantilever beam. When loading is required,

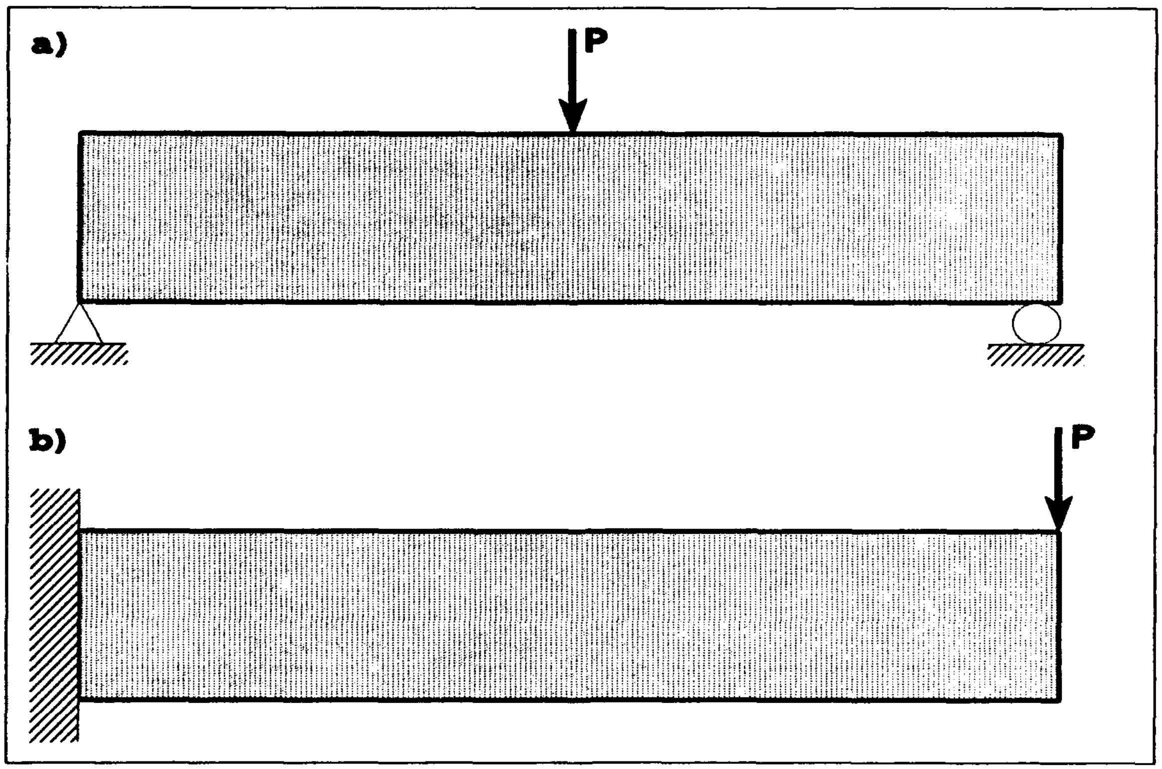

Figure 4 : a) Simply Supported Beam with Center Load b) Cantilever Beam with End Load.

the force is applied at the center of the simply supported beam and at the end of the cantilever beam as seen in Figure 4.

Material properties of the beams are as follows; elastic modulus E=1,000,000 Pa., shear modulus G=450,000 Pa., density ρ=10 g/cm^3, length of the beam L=60 m., height h=0.5 m. and applied force P=1 N.

First a free vibration analysis is performed, for both simply supported and cantilever beams. The analytical expression for the natural frequencies is as follows

$$\omega_i^2 = \frac{(k_i l)^4 EI}{\gamma l^4} \tag{35}$$

where γ is the mass per unit length of the beam and $k_i l$ are defined values for some beam types.

The results of first three natural frequencies are presented in Tables 1 and 2. As seen from the results, the finite element solution agrees with the exact solution very well. The mode shapes corresponding to first four natural frequencies are also plotted in Figures 5 and 6. These are expected mode shapes of the simply supported and the cantilever beams [Ref. 8].

TABLE 1 : NATURAL FREQUENCIES OF SIMPLY SUPPORTED BEAM

	ω_1	ω_2	ω_3
Exact Solution	0.00395	0.0158	0.0356
FEM Solution	0.00395	0.0158	0.0356
% Difference	-	-	-

TABLE 2 : NATURAL FREQUENCIES OF CANTILEVER BEAM

	ω_1	ω_2	ω_3
Exact Solution	0.00141	0.00883	0.0247
FEM Solution	0.00141	0.00883	0.0247
% Difference	-	-	-

Then static deflections of beams are computed. The analytical solutions for isotropic beams are [Ref. 9]

$$v=\frac{-Px}{48EI}(3L^2-4x^2) \qquad ,0<x<\frac{L}{2} \tag{36}$$

$$v=\frac{-Px^2}{6EI}(3L-x) \tag{37}$$

where Eq. (36) is for a simply supported beam and Eq. (37) is for a cantilever beam. Here "I" refers to the moment of inertia and L is the length of the beam. The results of both exact solution and finite element solution are plotted in Figure 7.

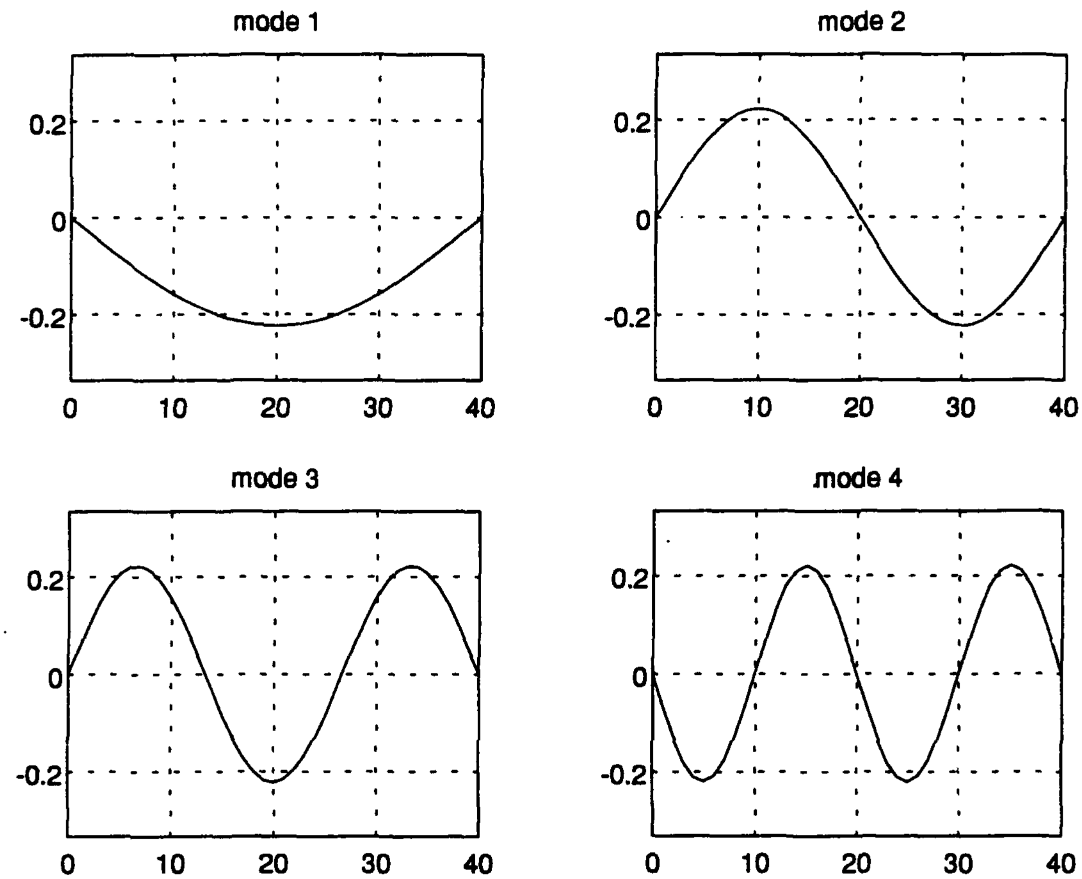

Figure 5 : Mode Shapes of the Simply Supported Beam

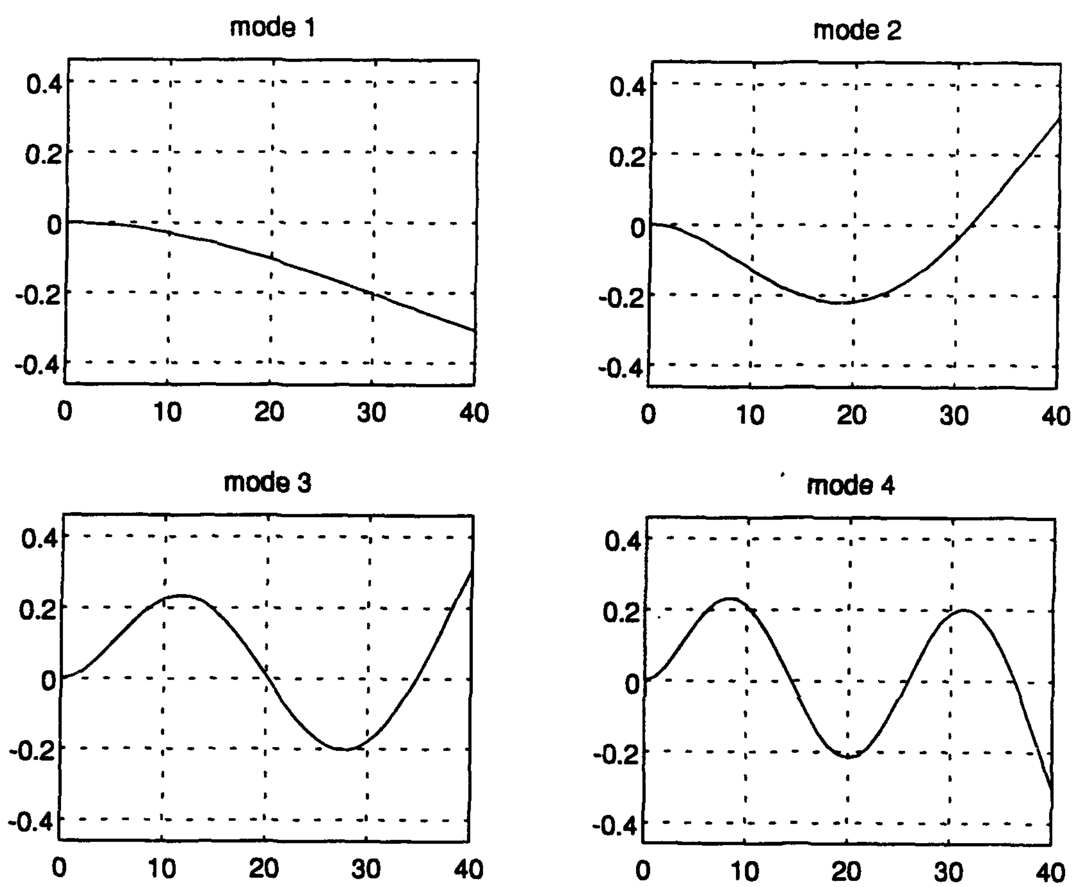

Figure 6 : Mode Shapes of the Cantilever Beam

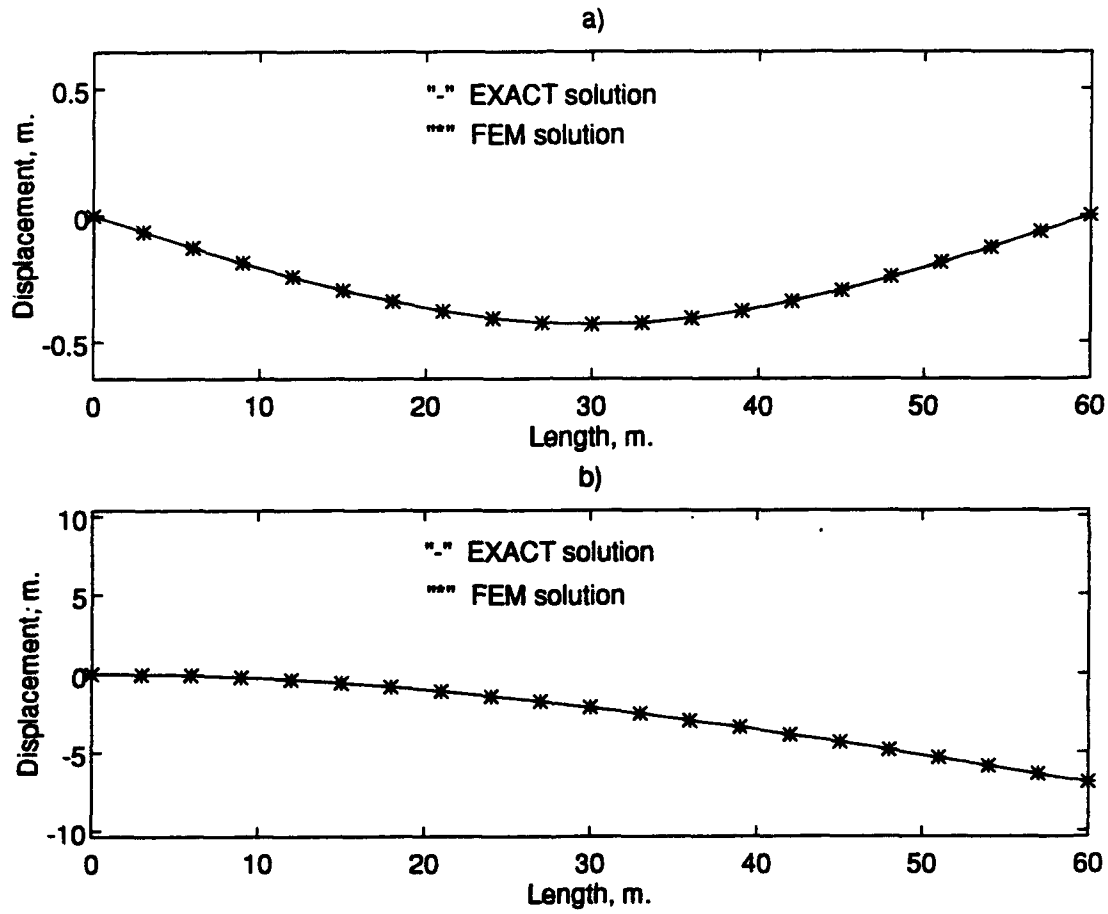

Figure 7 : Static Deflections of Isotropic Beams; a) Simply Supported Beam , b)Cantilever Beam

Finally a static analysis of a laminated composite beam with simple supports is performed. The analytical expression for the deflection of a simply supported beam, which is subject to three point bending, as shown in Figure 8, is as follows [Ref. 10]

$$V=-\frac{PL^2x}{48E_x^bI}\left[3-\left(\frac{2x}{L}\right)^2\right] \tag{38}$$

and the equivalent bending stiffness is defined as

$$E_x^bI=\sum_{k=1}^{N}E_1^kI^k \tag{39}$$

here E_x^b is the effective bending modulus of the beam, E_1^k is the modulus of the *k*th layer relative to the beam axis, I^k is the moment of inertia of the *k*th layer relative to the midplane, and N is the number of layers in the laminate. The vertical displacements obtained from the exact solution and the finite element solution are plotted together, for four and eight layered beams and for two different ratios of elastic modulii, in Figures 9 and 10.

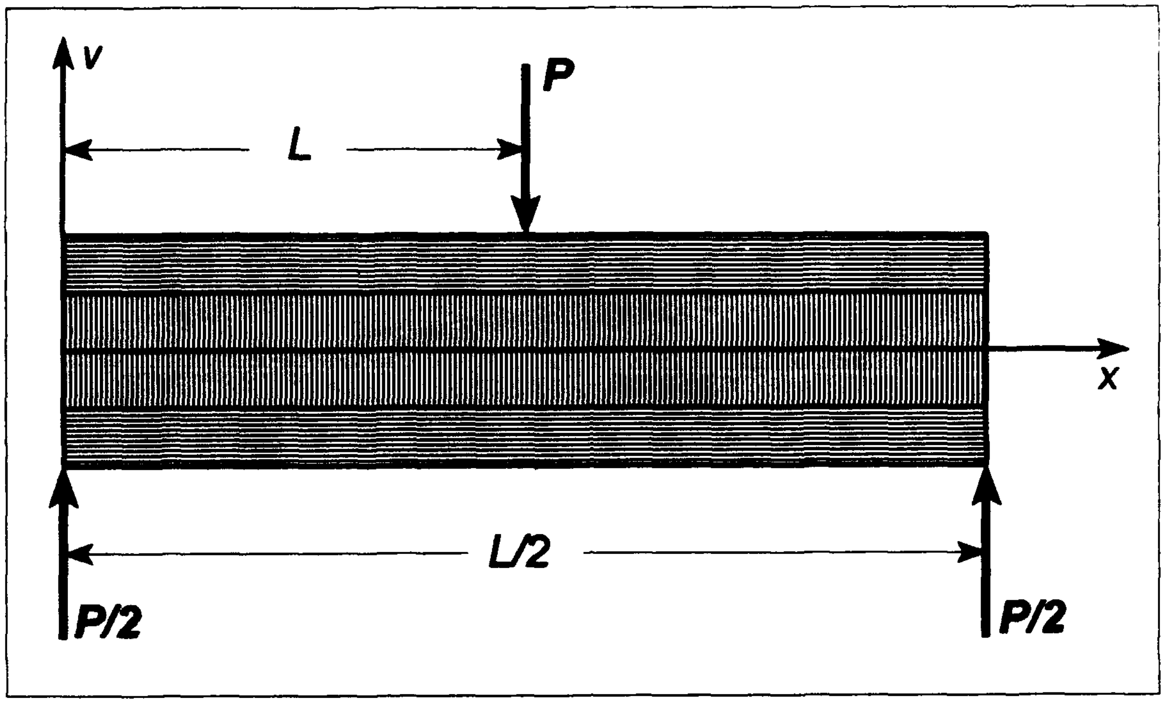

Figure 8 : Laminated Beam with Three Point Bending (Simply Supported).

In the static problems, as seen from the figure the exact solutions and the finite element solutions agree very well. From these verification examples, it can be shown that the beam element provides accurate results compared to exact solutions.

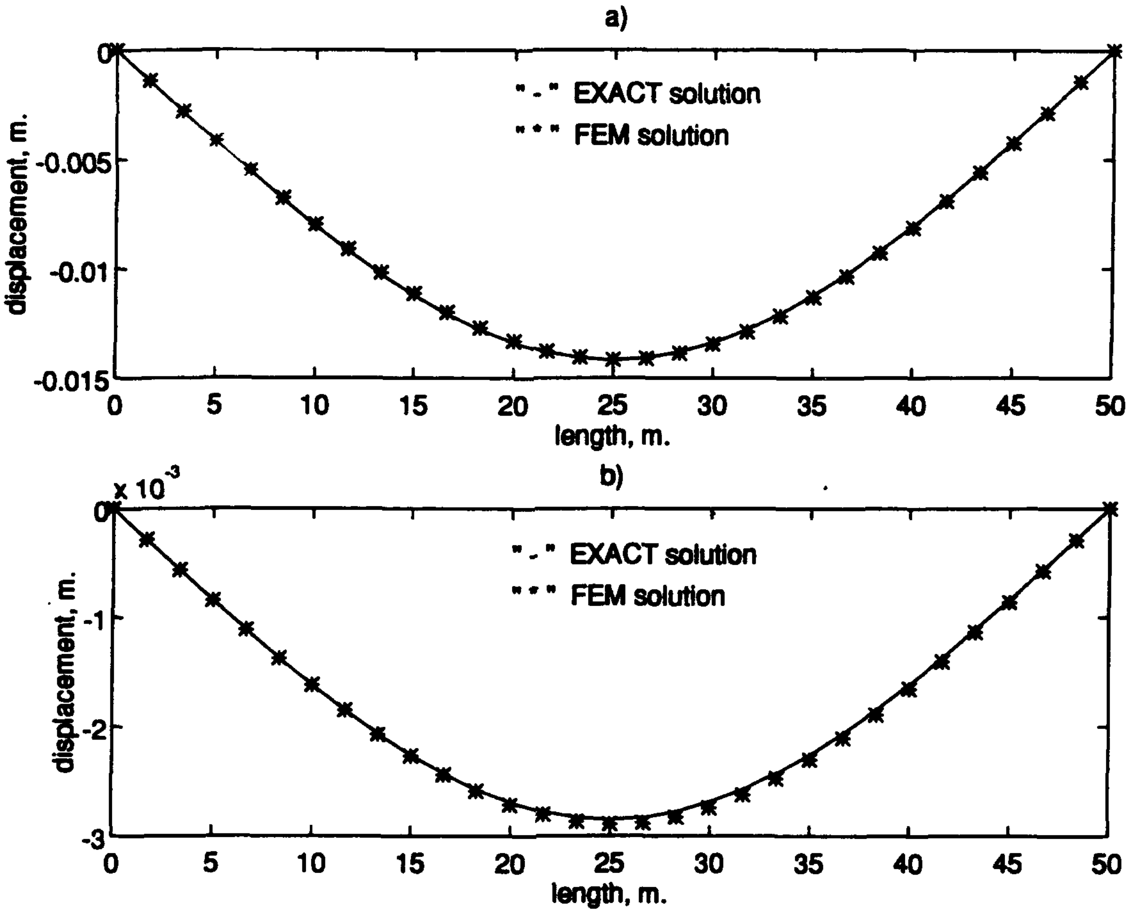

Figure 9 : **Static Deflections of the Laminated Composite Beam with 4 Layers a)for E_0/E_{90}=20, b) for E_0/E_{90}=100**

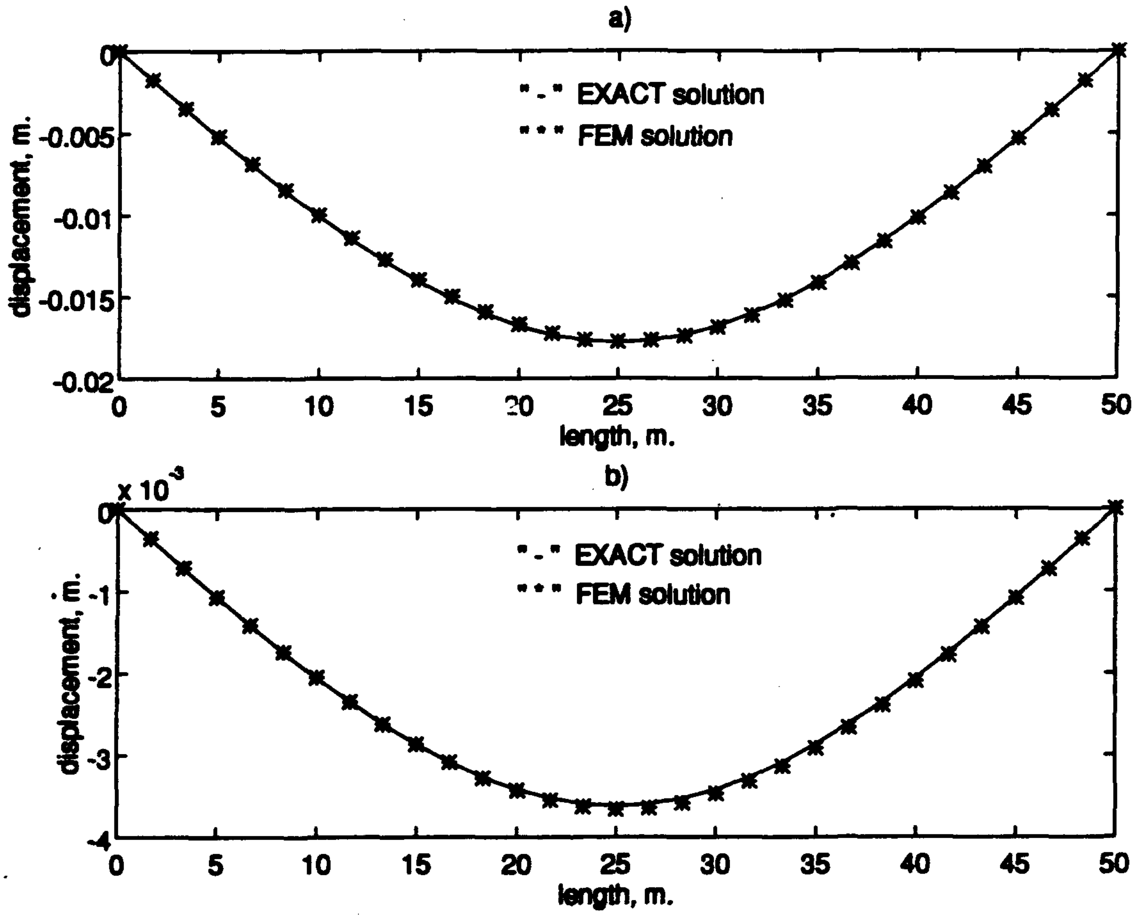

Figure 10 : **Static Deflections of the Laminated Composite Beam with 8 Layers**
a) for E_0/E_{90}=20, b) for E_0/E_{90}=100

B. NUMERICAL RESULTS

Dynamic analyses are performed for a simply supported composite beam with four layers, i.e. $[0/90]_s$, and with different crack sizes using contact-impact conditions. Calculations without contact-impact conditions are also made and compared to the case computed with contact-impact conditions. An impulsive load of a very short duration is applied at the center of the upper portion of a delaminated section and the displacement response of the beam is recorded for a period of time.

First displacements at the mid-points of upper and lower portions of a large crack are calculated and plotted in Figure 11 without using contact-impact conditions, where crack length is one third of the beam length. As seen from the figure, these two points overlap each other which is not admissible. Then for the same crack size, contact-impact conditions are applied and displacements of the beam is calculated and plotted. Figure 12 shows the displacements of the beam just before contact while the displacements at the time of contact is shown in Figure 13. The displacements just after contact is shown in Figure 14. Also the responses of the mid-points and quarter-points are plotted in Figures 15 and 16, respectively, where quarter-points represent the points which are located in the middle between the crack tip and the crack midpoint for both upper and lower portions. Later, the crack size is reduced to one thirtieth of the beam length and the displacements of mid-points and quarter-points are plotted in Figures 17 and 18, respectively. As seen in these results, the upper and lower portions never penetrate each other. This indicates that the application of contact-impact conditions works well in simulating delaminated beams. Finally, the response of a beam without a crack is computed and plotted in Figure 19 for comparison purpose. Responses of all these cases are different each other. They may give an idea how an embedded crack affects the behavior of beams.

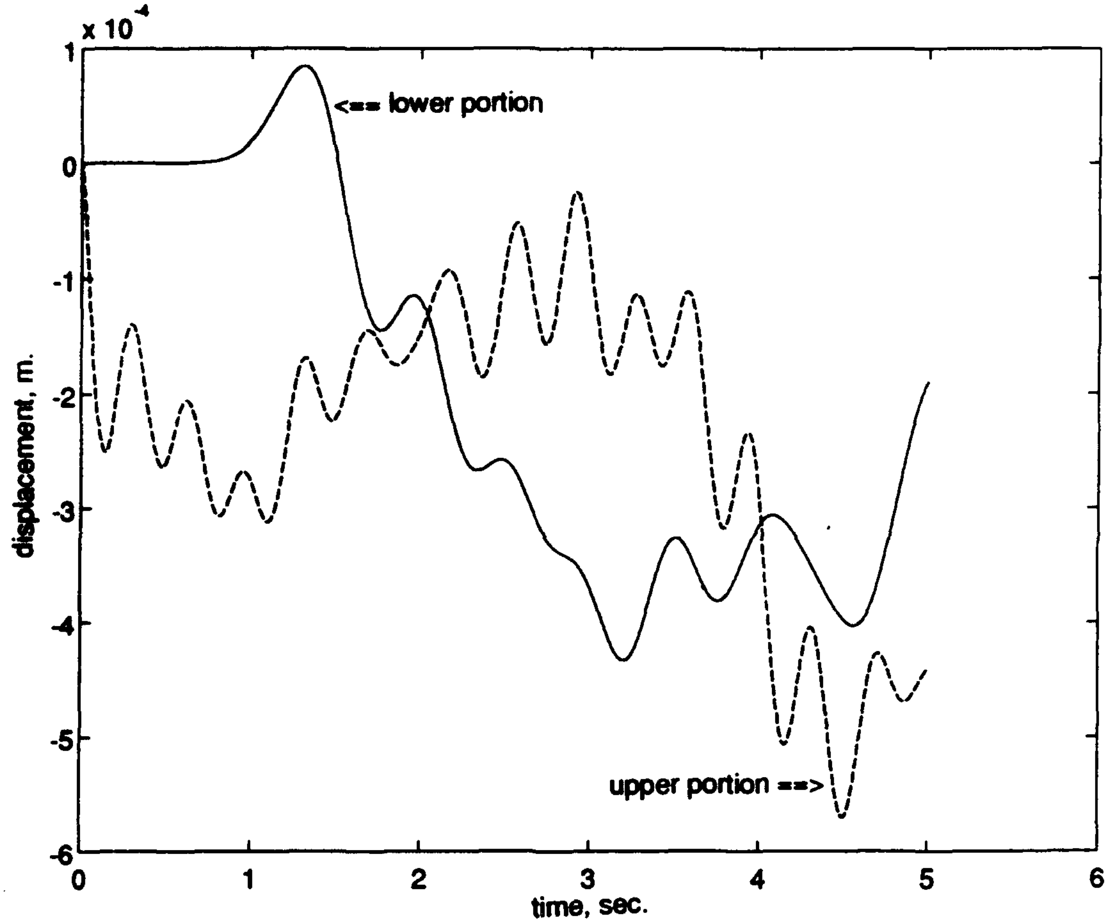

Figure 11 : Displacements of Mid-Points without Contact-Impact Conditions (Large Crack Size)

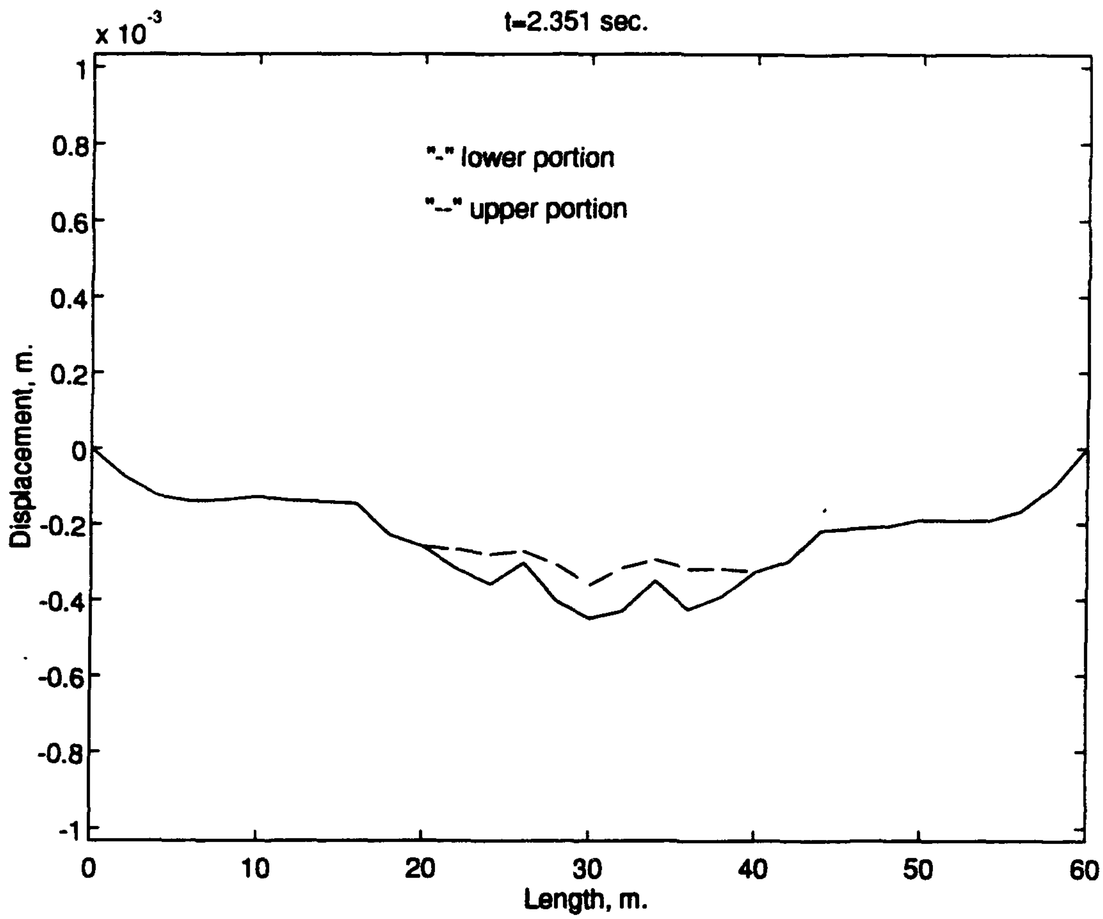

Figure 12 : Displacement of Beam (before Contact)

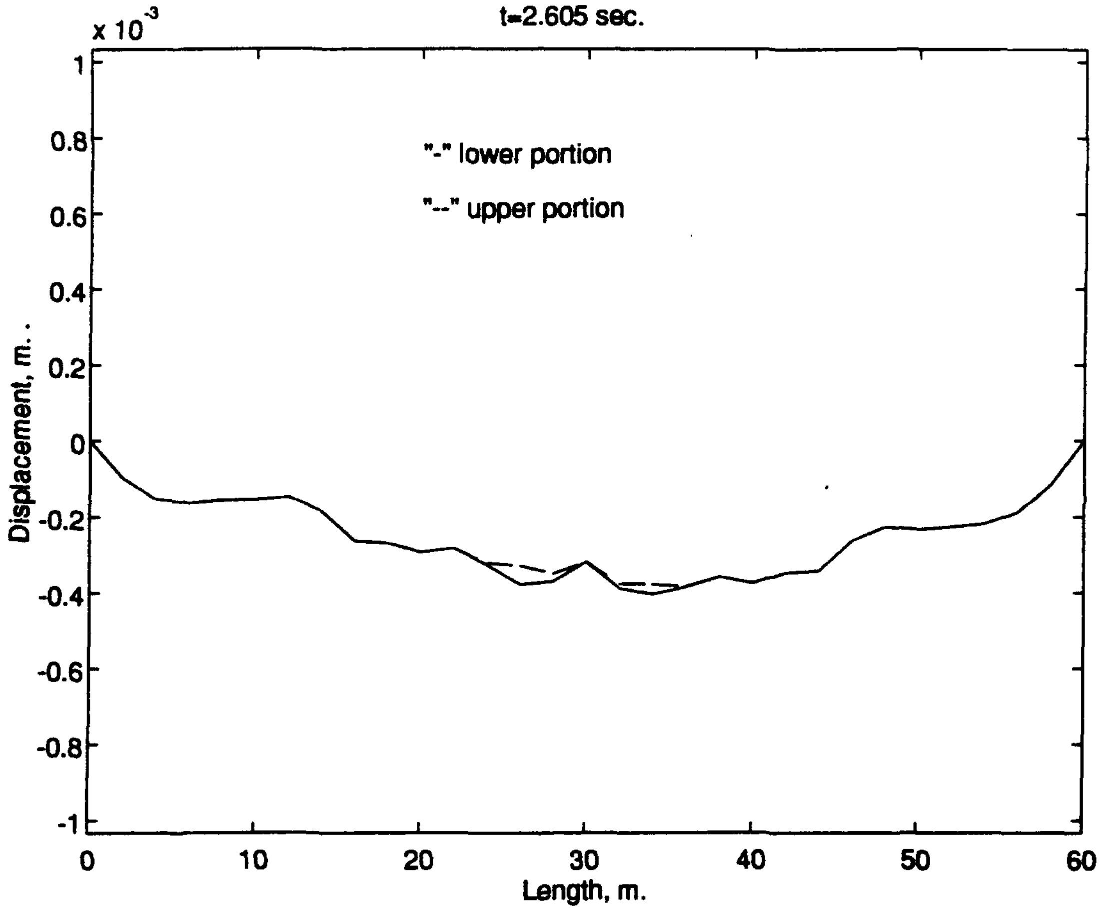

Figure 13 : Displacement of Beam (at Contact)

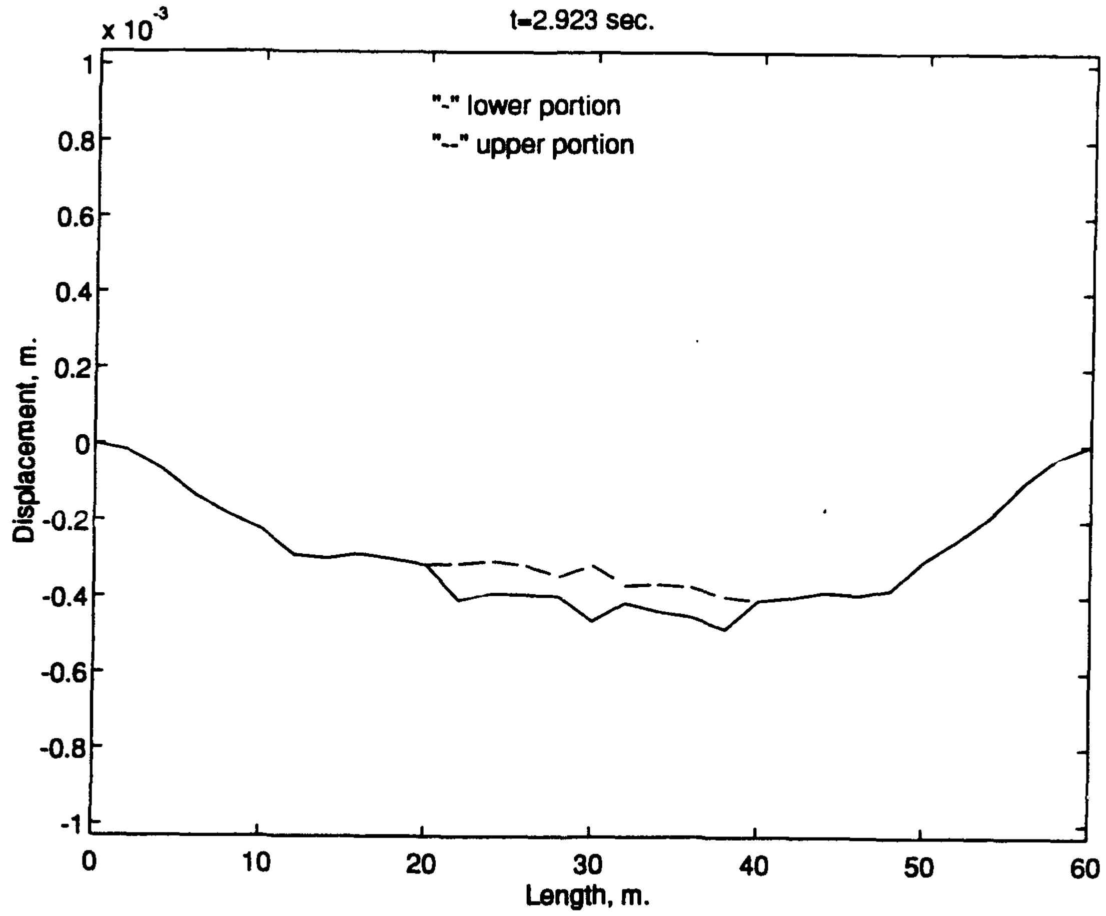

Figure 14 : Displacement of Beam (after Contact)

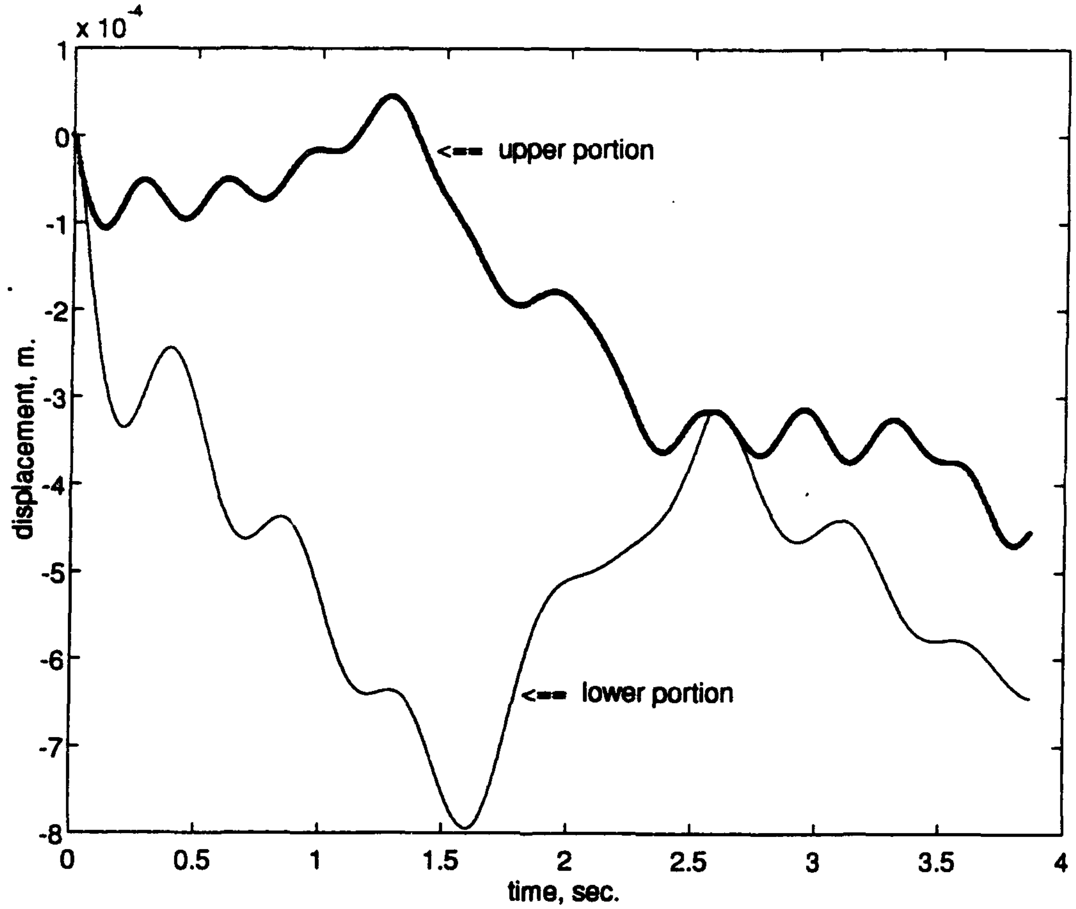

Figure 15 : Displacements of Mid-Points with Contact-Impact Conditions (Large Crack Size)

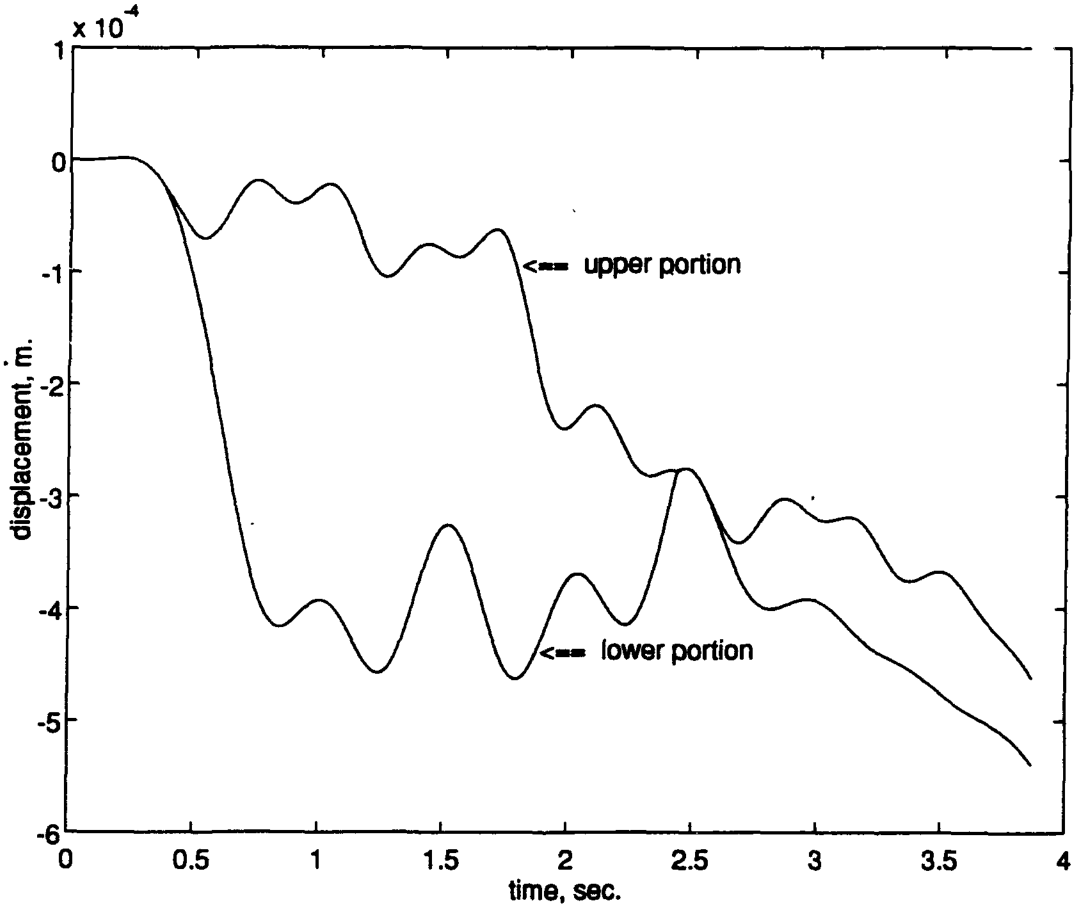

Figure 16 : Displacements of Quarter-Points with Contact-Impact Conditions (Large Crack Size)

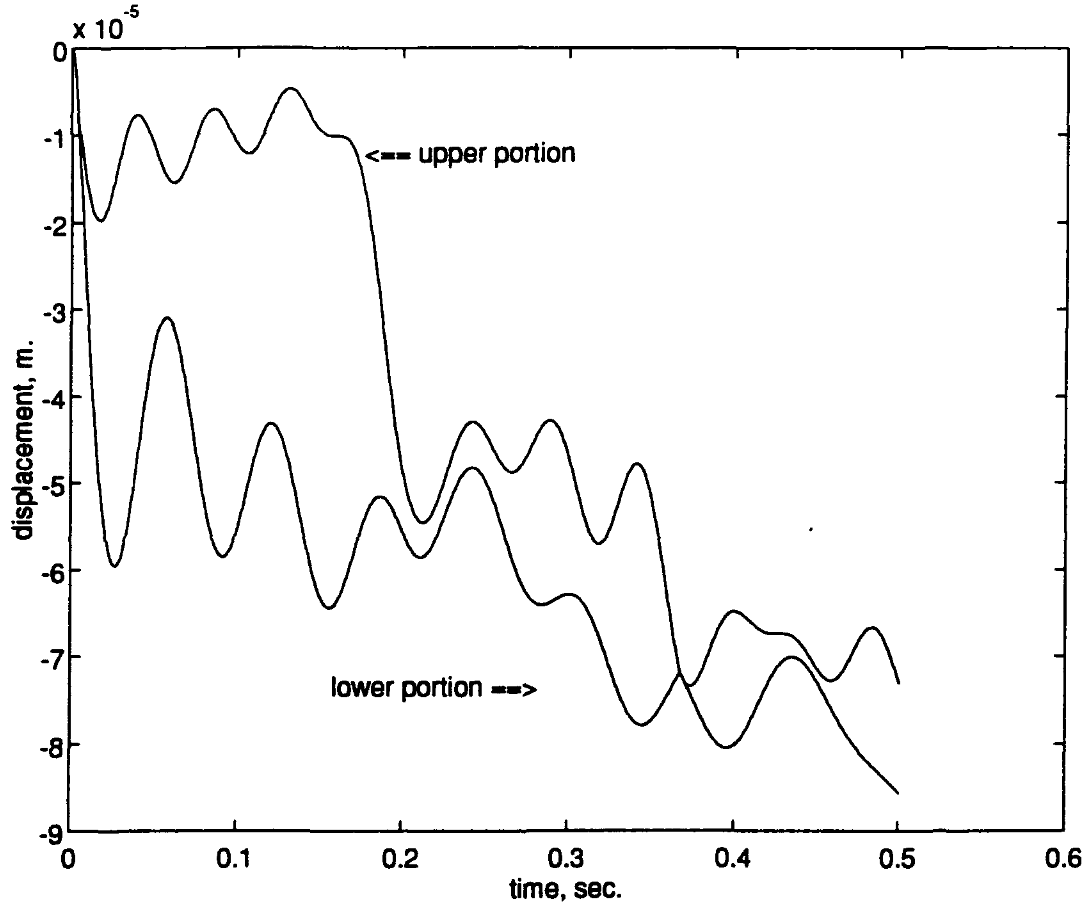

Figure 17 : Displacements of Mid-Points with Contact-Impact Conditions (Small Crack Size)

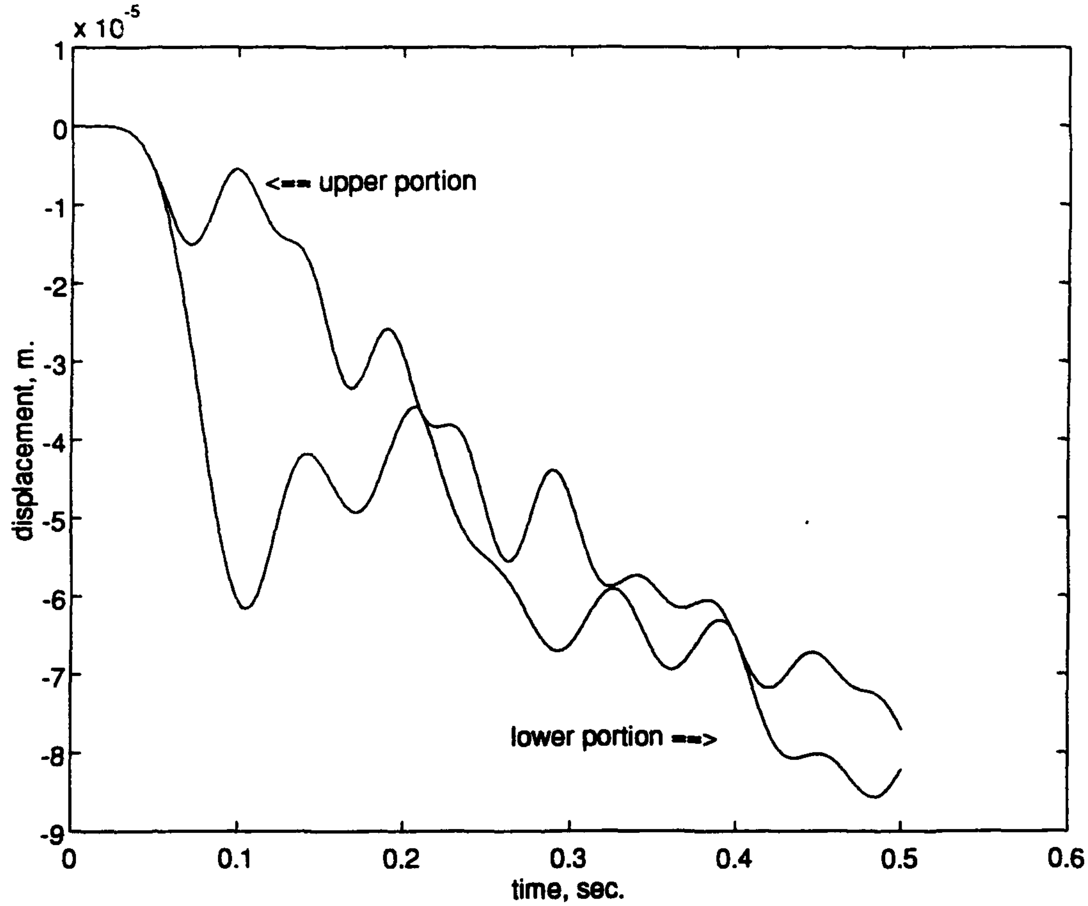

Figure 18 : Displacements of Quarter-Points with Contact-Impact Conditions (Small Crack Size)

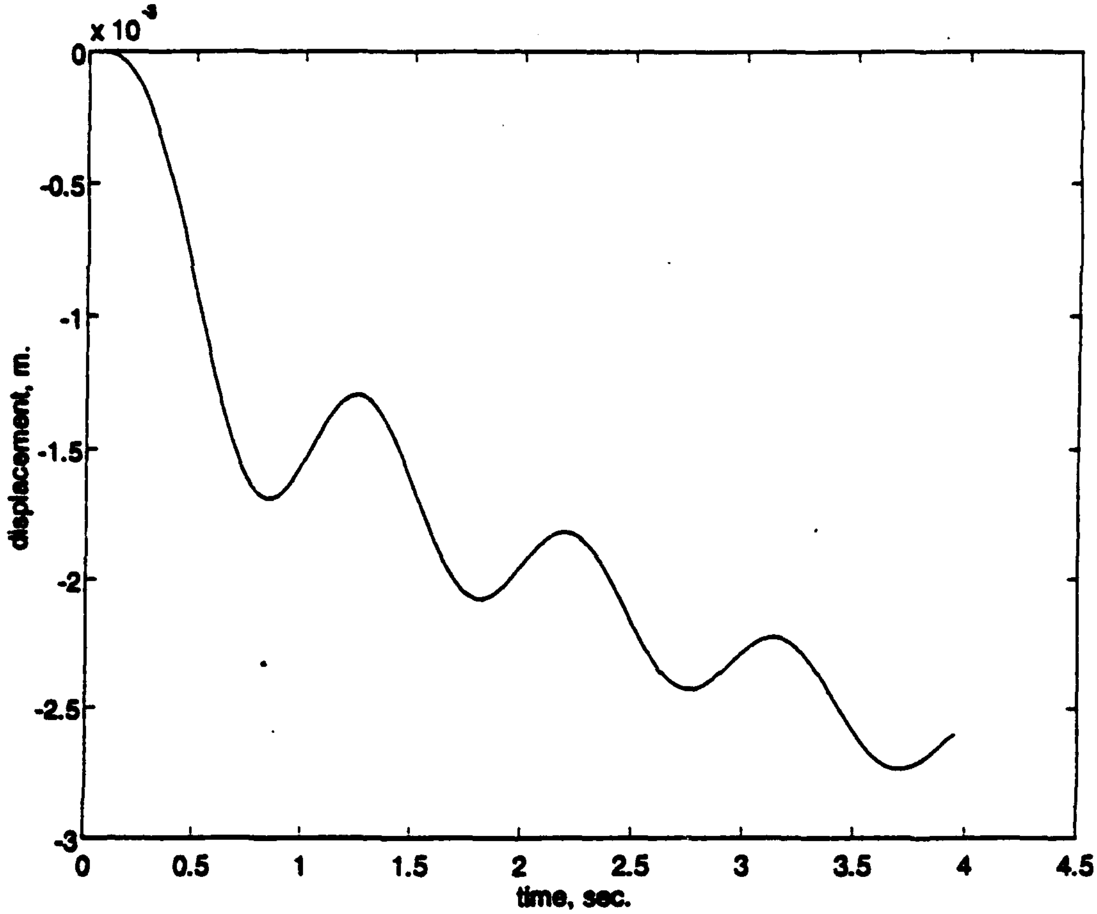

Figure 19 : Displacements of Mid-Point of a Beam without Crack

IV. CONCLUSIONS AND RECOMMENDATIONS

The beam element used in this study is useful to model a crack with contact-impact conditions. Application of contact-impact conditions constrains the vertical displacements at the crack in a way that the points on lower and upper portions of the crack hit each other without penetration.

Computational time depends on the number of degrees of freedom. The technique, which models the laminated composite beam using a single beam element, reduces the degrees of freedom to save time during calculations.

When an impulsive load of a short duration is applied, the beam responds in different ways depending on whether there is a crack inside it and what the crack size is. Examining these results gives an idea about existence of an embedded crack as well its size and location.

For further studies, delaminated sections may be refined to see the minimum crack size that can be detected using this procedure. Also, the Fast Fourier Transform can be applied to measure the frequency response of a delaminated beam. Comparing the natural frequencies may give an idea about the crack size. In this study, a crack is assumed to be along the middle axis of the beam and the load is applied at the center of the beam. Locations of cracks and loads may be changed and the responses can be calculated for different cases.

Some experiments are being performed to see the response of beams under impulsive loading. The results obtained from the procedure described in this study may be compared with those experimental results for verification.

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